On automorphism groups of power semigroups over numerical semigroups

Dein Wong

Abstract. A numerical semigroup S is a cofinite subsemigroup of $\mathbb{N}$, where $\mathbb{N}$ is the additive monoid of non-negative integers. Denote by $\mathcal{P}_{\mathrm{fin}}(S)$ the semigroup consisting of all non-empty finite subsets of S endowed with the operation of setwise addition defined by

$$X + Y = \{x + y : x \in X, y \in Y\}, \qquad \text{for all } X, Y \in \mathcal{P}_{\mathrm{fin}}(S).$$

We call $\mathcal{P}_{\mathrm{fin}}(S)$ the finitary power semigroup of S. When $0 \in S$ (and hence S is a numerical monoid), the family $\mathcal{P}_{\mathrm{fin},0}(S)$ of all finite subsets of S containing 0 is a submonoind of $\mathcal{P}_{\mathrm{fin}}(S)$, whose identity element is the singleton $\{0\}$; we call $\mathcal{P}_{\mathrm{fin},0}(S)$ the reduced finitary power monoid of S.

Tringali and Yan have recently proved in [J. Combin. Theory Ser. A 209 (2025)] that the only non-trivial automorphism of $\mathcal{P}_{\mathrm{fin},0}(\mathbb{N})$ is the involution $X \mapsto \max X - X$. In this talk, we show how Tringali and Yan's result can be used to determine the automorphism group of the finitary power semigroup of an arbitrary numerical semigroup S. More precisely, if S is the set of all integers larger than or equal to a fixed $k \in \mathbb{N}$, then the only non-trivial automorphism of $\mathcal{P}_{\mathrm{fin}}(S)$ is the involution $X \mapsto \max X - X + \min X$; otherwise, $\mathcal{P}_{\mathrm{fin}}(S)$ has only the trivial automorphism.

About the speaker. Dein Wong was born in February 1965. He graduated from the University of Science and Technology of China in 1998, where he obtained his PhD in mathematics. He is currently a professor at the China University of Mining and Technology, where he also serves as Director of the Institute of Fundamental Mathematics.

His research interests include classical groups, finite simple groups of Lie type, Lie algebras, and algebraic graph theory. He has published 138 papers as first or corresponding author in T-category journals of the Chinese Mathematical Society and has been principal investigator of four General Projects funded by the National Natural Science Foundation of China.

References

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